DYNAMIC ELASTIC MODULUS AND DAMPING COEFFICIENT OF SOME INDIAN TIMBERS

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(*Received July 19, 1963*)

ABSTRACT. The dynamic elastic modulus E_d and damping coefficient in terms of $1/Q$, which is also related to logarithmic decrement δ, have been measured parallel to grain on thirteen kinds of Indian timber employing a flexural vibration method. The static elastic modulus E_s has also been determined on the same samples.

The dynamic modulus E_d varies from 0.80×10^{11} to 1.61×10^{11} dynes/cm² and E_s the static modulus from 0.66×10^{11} to 1.42×10^{11} dynes/cm². E_d averages about 15 per cent higher than E_s while δ ranges from 0.028 to 0.053. Results show that in general δ increases with E_d/E_s.

INTRODUCTION

Considerable amount of indigenous timbers is being used for the construction of aircrafts, automobiles, bridges and similar other structures which are subject to dynamic forces. But as yet no systematic information is available on the dynamic elastic and damping properties of these materials. A knowledge of the dynamic elastic and damping characteristics of timbers is helpful in making an intelligent and economic use of the materials in all technical applications where vibrations must be considered. From an engineering point of view, in structural components where vibration is a hazard, the use of materials which have a high damping capacity is preferred if other strength properties are satisfactory. The damping capacity of wood is greater than for most other structural materials particularly the metals (Brown, Panshin and Forsaith, 1952).

The present preliminary investigation relates to some indigenous timbers of common use. Only 13 kinds of timber were selected for study from those listed in IS 399–1952 "Indian Standard Classification of Commercial Timbers and their Zonal Distribution." The timber specimens were procured in the log form (about 3 ft. long) from the forest departments of the various states in India. Only one log of each type was obtained. No attempt was made to study the variations of the properties in different parts of the same tree, or from tree to tree of the same type grown in different regions, nor even the properties in different directions. No exhaustive statistical analysis of the data is, therefore, made; only the mean and standard errors are shown.

The dynamic elastic modulus and the damping coefficient were determined parallel to the grain at about 12% moisture content by a flexural vibration method. These were compared with the static modulus determined on the same samples by the loaded beam method.

THEORETICAL CONSIDERATIONS

Dynamic Modulus of Elasticity (Young's)

The method is based on the vibration of a rectangular bar in the free-free mode i.e. the ends of the bar are left free. It has been shown that in this mode of vibration at the fundamental resonance frequency the nodal points are situated at distances 0.224 times the length of the bar from either ends. The frequency of the vibration in such a mode has been worked out (Lord Rayleigh, 1926) as

$$f = \frac{kcm^2}{2\pi l^2}$$

where k = radius of gyration of the section about an axis perpendicular to the plane of bending.
c = velocity of propagation of sound,
m = a constant = 4.730 for the fundamental,
l = length of the bar.

Since $c = \left[\frac{E_d}{\rho}\right]^{\frac{1}{2}}$

where E_d = dynamic modulus of elasticity,
ρ = density;

$$E_d = \frac{4\pi^2 l^4 f^2 \rho}{m^4 k^2}$$

For a bar of rectangular cross-section $k = \frac{t}{(12)^{\frac{1}{2}}}$, where t is the thickness, so that

$$E_d = \frac{48\pi^2 l^4 f^2 \rho}{m^4 t^2} T,$$

where T is a correction factor introduced (Timoshenko 1921, 1922) to account for the effect of rotary inertia and moment of shear. T has been shown to depend on $\frac{k}{l}$ and Poisson's ratio μ. Values of T have been computed for the fundamental frequency, assuming $\mu = \frac{1}{3}$ using the equation (Pickett, 1945)

$$T = 1 + 88.12\left(\frac{k}{l}\right)_2 - \frac{1572\left(\frac{k}{l}\right)^4}{1 + 92.61\left(\frac{k}{l}\right)^2} - 125\left(\frac{k}{l}\right)^4$$

This can be made nearly equal to unity by making $\frac{k}{l}$ small. In the actual samples chosen the thickness was small compared to the length that is $\frac{t}{l} = 0.025$ so that $T = 1.005$ for $\mu = \frac{1}{3}$. The value of μ (longitudinal) for wood varies between $\frac{1}{4}$ to $\frac{1}{2}$ (Hearmon, 1953). The correction therefore amounts to only 0.5% and so has been neglected.

Damping Coefficient

The energy required to maintain a bar in sustained vibration is dissipated in overcoming the internal friction of the bar in which case it is converted into heat and secondly, in transferring energy to the external surrounding. The damping capacity ΔW is the energy per unit volume per cycle used in overcoming the internal friction. Following electrical analogy the Q of a specimen is given as (Obert and Duvall, 1941),

$$Q = \frac{2\pi W}{\Delta W},$$

where W is the total energy of vibration per unit volume per cycle. Q can be measured from the sharpness of resonance curve; if f_0 is the resonant frequency and Δf the width of the resonance curve in cycles/sec at $\frac{1}{2^{\frac{1}{2}}}$ of the maximum amplitude then,

$$Q = \frac{f_0}{\Delta f}.$$

Similarly in terms of the logarithmic decrement δ of a free vibration whose amplitude decreases exponentially

$$Q = \frac{\pi}{\delta}.$$

In the present investigation it has been found convenient to evaluate $\frac{1}{Q} = \frac{\Delta f}{f_0}$ and also the logarithmic decrement $\delta = \frac{\pi}{Q}$.

Static Modulus of Elasticity

Under static bending in the case of a simple end-supported beam of a rectangular cross-section carrying a centre load, the modulus of elasticity E_s is given by (Brown etc., 1952)

$$E_s = \frac{PL^3}{48yI},$$

where P = Load applied at centre of beam,

L = Span,

y = Deflection due to load P,

I = Moment of inertia of the beam

$= b\,t^3/12$, where b = breadth of beam, and

t = thickness of beam.

Substituting the value of I

$$E_s = \frac{PL^3}{4ybt^3}.$$

EXPERIMENTAL

The logs of wood in air-dry condition were sawn into 2ft. × 2 ins × 2 ins sticks in accordance with the sectioning scheme shown in Fig. 1. About 20-25 samples having length 10 inches along the grain, breadth 0.5 inch and thickness 0.25 inch,

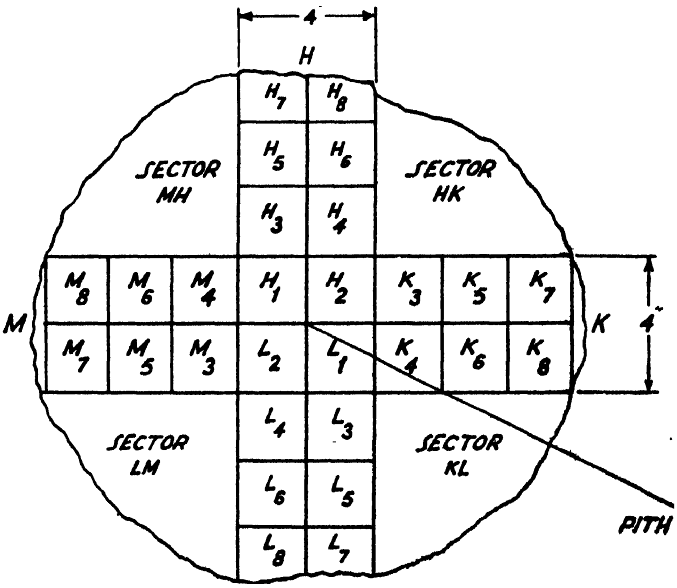

Fig. 1. Sectioning Scheme of Sawing and Sampling a Timber Log.

were prepared out of these sticks cut from a log. Prior to test these samples were conditioned to a moisture content of about 12 per cent by keeping them over a saturated solution of sodium chloride at room temperature for about two weeks till there was no change in weight.

The apparatus used is shown in Fig. 2.

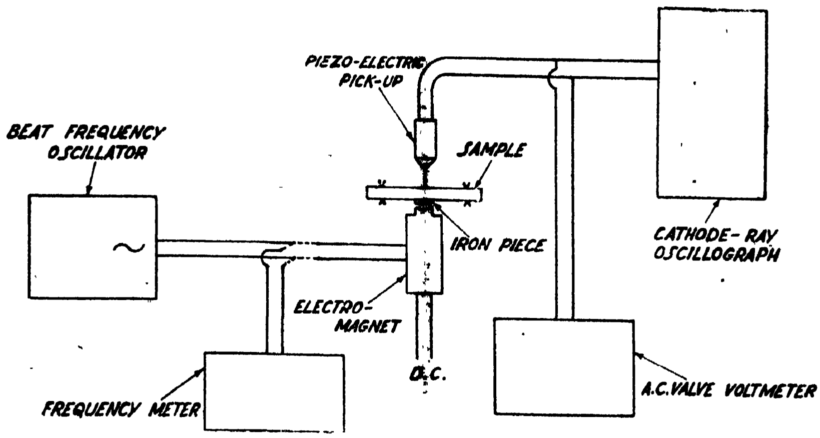

Fig. 2. Schematic Diagram of the Apparatus for the Measurement of Dynamic Young's Modulus and Damping Coefficient of Timber in Flexural Vibration.

A small rectangular piece of soft-iron sheet weighing about 0.27 gm. was tached firmly at the centre of the sample. The sample was mounted on two horizontal knife-edge supports at the nodal points, the two ends being left free. The knife-edges are embedded in heavy iron blocks previously adjusted for parallelism and separated by a distance equal to $0.552l(= l - 2\times 0.224l)$. To ensure that the sample properly rested on the lower knife edges and was not bodily shifted while vibrating, masses of approximately 120 gms. each were suspended from two knife edges placed exactly above the lower knife edges at the nodal points. The effect of these masses on thevib ration of the sample was found to be insignificant since they were suspended at the nodal points.

The electro-magnetic driver was positioned just below the centre of the sample under the soft-iron piece fixed to the sample. The distance between the electromagnet and the iron piece fixed to the sample was about 1 mm. The electromagnet was energised by a beat frequency oscillator, and the sample was made to vibrate to resonance at the fundamental by varying the frequency of the oscillator.

The amplitude of vibration was observed by means of a piezoelectric pick-up connected to a valve voltmeter and to a cathode-ray oscillograph. At resonance the maximum wave-amplitude on the oscillograph screen as well as the voltage indicated on the valve voltmeter were noted. The resonance frequency f_0 was measured on a frequency meter. All the samples were tested under conditions to give the same amplitude of vibration as measured on the valve voltmeter.

The two frequencies f_1 and f_2 at which the pick-up voltage falls to $1/2^{\frac{1}{2}}$ of its maximum value were found. As f_1 and f_2 were near, the frequency scale of the oscillator was enlarged by arranging a condenser across the zero adjusting condenser of the beat frequency oscillator (Obert and Duvall, 1941). This gives Δf.

TABLE I

Results of Measurement of Dynamic and Static Young's Moduli, 1/Q & δ of Indian Timbers (Parallel to Grain).

Sl. No.	Kind of timber: Botanical Name (Common Name)	Moisture content %	No. of test pieces taken	Density ρ = Mass at test / Vol. at test gms/cm³ Mean	Resonance Frequency of gmg/cm³ c.p.s. Range	Young's Moduli: E_d Dynamic (Adiabatic) $10''$ dynes/cm² Mean ± S.E.	Young's Moduli: E_s Static (Isothermal) $10''$ dynes/cm² Mean ± S.E.	Ratio $\gamma = \frac{E_d}{E_s}$ Mean	Damping Coefficient $\frac{1}{Q} = \frac{\Delta f}{f_0}$ Mean	Logarithmic Decrement $\delta = \pi \frac{1}{Q}$ Mean ± S.E.
1.	Tectona grandis (Teak)	10.9	25	0.594	410–525	1.21 ± 0.03	1.09 ± 0.03	1.12	0.010	0.030 ± 0.001
2.	Shorea robusta (Sal)	11.7	21	0.793	420–490	1.61 ± 0.03	1.42 ± 0.04	1.14	0.009	0.028 ± 0.002
3.	Albizzia lebbeck (Kokko)	9.7	22	0.654	446–540	1.53 ± 0.04	1.38 ± 0.03	1.11	0.010	0.031 ± 0.002
4.	Terminalia tomentosa (Laurel)	11.9	22	0.758	325–401	0.89 ± 0.03	0.71 ± 0.03	1.26	0.017	0.053 ± 0.001
5.	Adina cordifolia (Haldu)	13.2	24	0.636	389–520	1–18 ± 0.03	1.04 ± 0.03	1.14	0.010	0.030 ± 0.002
6.	Pinus longifolia (Chir)	13.3	24	0.614	483–560	1.54 ± 0.03	1.41 ± 0.03	1.09	0.014	0.045 ± 0.001
7.	Cedrus deodara (Deodar)	13.8	24	0.601	350–524	1.08 ± 0.04	0.90 ± 0.04	1.20	0.014	0.045 ± 0.002
8.	Picea morinda (Spruce)	12.4	24	0.461	427–571	1.09 ± 0.04	0.95 ± 0.04	1.16	0.011	0.036 ± 0.001
9.	Terminalia paniculata (Kindal)	14.4	22	0.884	387–486	1.44 ± 0.05	1.26 ± 0.04	1.14	0.013	0.040 ± 0.002
10.	Langerstroemia lanceolata (Benteak)	14.0	23	0.704	370–480	1.14 ± 0.04	0.93 ± 0.04	1.24	0.014	0.043 ± 0.001
11.	Dalbergia latifolia (Rosewood)	10.7	21	0.788	385–441	1.26 ± 0.02	1.10 ± 0.02	1.15	0.011	0.035 ± 0.001
12.	Pinus excelsa (Blue Pine)	11.7	25	0.427	373–559	0.81 ± 0.03	0.66 ± 0.03	1.23	0.014	0.043 ± 0.001
13.	Artocarpus integrifolius (Kathal)	12.9	24	0.508	374.460	0.80 ± 0.02	0.67 ± 0.02	1.20	0.012	0.039 ± 0.001

S.E. = Standard Error.

From the above observations the dynamic elastic modulus E_d, the damping coefficient in terms of $1/Q$ and the logarithmic decrement δ were calculated.

For the measurement of E_s by the static method, the sample was supported on two knife-edges with a span of 25.0 cms. and loaded at the centre. The load was increased to 2 Kgms in steps of 500 gms and the maximum deflections were noted by a gauge with a least count of 1 mil. (0.001″). The deflections were also noted while unloading. The mean deflection for 500 gms was found from which E_s was calculated.

Finally the moisture content was determined on six samples selected from the 20-25 samples, by the oven-dry method.

RESULTS

The average values of the density ρ, dynamic and static Young's moduli E_d and E_s, their ratio $\gamma = E_d/E_s$, the damping coefficient $1/Q$ and the logarithmic decrement δ are entered in Table I. As an estimate of the dispersion of the observed E_d and E_s, their standard errors are also shown.

DISCUSSION OF RESULTS

The results show that the dynamic elastic modulus is about 10–20% higher than the static value. This is in agreement with the findings of other workers quoted in literature (Brown, etc., 1952)(Kuenzi, 1952) and to be expected on theoretical considerations since the static value measures the isothermal modulus and the dynamic value at the frequencies involved (300-600 c/s) measures the adiabatic modulus (Mason, 1958).

The logarithmic decrement δ varies between 0.028 to 0.053 which is of the same order as observed by other investigators quoted in literature (Brown, etc, 1952)(Hearmon, 1953). The δ values plotted against $\gamma = E_d/E_s$ shows a general trend of increase of δ with γ. This is to be expected since both δ and γ are concerned with the frictional heat losses within the sample.

ACKNOWLEDGEMENTS

Grateful acknowledgements are due to Dr. P. S. Srinivasan for initiating this research problem and for his initial guidance in the work. We also acknowledge our thanks to Dr. J. N. Nanda, Director, Defence Research Laboratory (Materials), Kanpur for his keen interest and permission to publish this paper.

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